

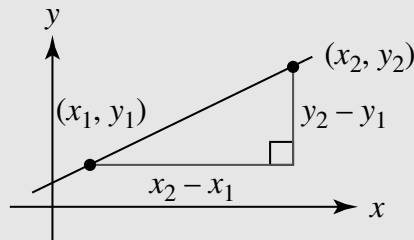
Topic F: Line graphs



This topic recaps how you can calculate key properties of straight line graphs when given two points on the line, in particular: the gradient, the length of a line segment, the midpoint of a line segment, the equation of the perpendicular bisector of a line segment, and the equation of the line. The gradient of a line is a measure of how steep it is.

The gradient, m , of a line between two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

Key point



Example 1

Calculate the gradient of the line through the points $A(1, -6)$ and $B(-5, 2)$

$$\begin{aligned} m &= \frac{2 - (-6)}{(-5) - 1} \\ &= \frac{8}{-6} \\ &= -\frac{4}{3} \end{aligned}$$

The line has a negative gradient so slopes down from left to right.

Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ with $x_1 = 1$, $x_2 = -5$ and $y_1 = -6$, $y_2 = 2$

Find the gradient of the line through each pair of points.

Try It 1

- a** $(1, 7)$ and $(4, 8)$ **b** $(8, -2)$ and $(4, 6)$ **c** $(-8, 7)$ and $(-4, -7)$



You also can find the length of a line segment between two points using Pythagoras' theorem.

Key point

The length of the line segment, d , between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 2 Calculate the exact distance between the point (5, 1) and (6, -4)

Use

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

with $x_1 = 5$, $x_2 = 6$ and $y_1 = 1$, $y_2 = -4$

Leave answer as a surd since this is exact.

$$d = \sqrt{(6-5)^2 + (-4-1)^2}$$

$$= \sqrt{1^2 + (-5)^2}$$

$$= \sqrt{26}$$

Calculate the exact distance between each pair of points. **Try It 2**

a (5, 2) and (7, 4) **b** (6, -4) and (-3, -1) **c** ($\sqrt{2}$, 4) and ($4\sqrt{2}$, -5)

The midpoint of a line segment is half-way between the points at either end.

The midpoint of the line segment from (x_1, y_1) to (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ **Key point**

Example 3 The points A and B have coordinates $(-4, -9)$ and $(6, -2)$ respectively. Find the midpoint of AB

$$\begin{aligned} \text{Midpoint} &= \left(\frac{(-4)+6}{2}, \frac{(-9)+(-2)}{2}\right) \\ &= \left(\frac{2}{2}, \frac{-11}{2}\right) \\ &= (1, -5.5) \end{aligned}$$

Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
with $x_1 = -4, x_2 = 6$ and
 $y_1 = -9, y_2 = -2$

Calculate the midpoint of the line segment between each pair of points. **Try It 3**

a $(1, 9)$ and $(2, 5)$ **b** $(-2, 3)$ and $(-5, -7)$ **c** $(6.4, -9.3)$ and $(-2.6, -3.7)$

You need to be able to work with equations of straight lines.

The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y -intercept.

Key point

Example 4

Work out the gradient and the y -intercept of each of these lines.

a $y = \frac{1}{2}x + 4$ **b** $y + x = 5$ **c** $-2x + 3y + 7 = 0$

a Gradient = $\frac{1}{2}$ and y -intercept = 4

b $y = 5 - x$

So gradient = -1 and y -intercept = 5

c $3y = -7 + 2x$

$y = -\frac{7}{3} + \frac{2}{3}x$

So gradient = $\frac{2}{3}$ and y -intercept = $-\frac{7}{3}$

Since $y = mx + c$ where m is the gradient and c is the y -intercept.

Rearrange to make y the subject.

Rearrange to make y the subject.

Work out the gradient and the y -intercept of each line.

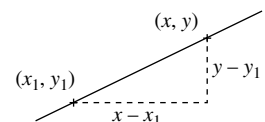
Try It 4

a $y = 8 - 2x$ **b** $2y + x = 3$ **c** $6x - 9y - 4 = 0$

You can write the gradient of a line in terms of a known point on the line (x_1, y_1) , the general point (x, y) , and the gradient, m .

$m = \frac{y - y_1}{x - x_1}$ or alternatively $y - y_1 = m(x - x_1)$

Gradient = $m = \frac{y - y_1}{x - x_1}$



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The equation of the line with gradient m through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$ **Key point**

If you have the coordinates of two points on a line then you can find the equation of the line. First use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line then substitute into $y - y_1 = m(x - x_1)$. Sometimes you will then need to rearrange the equation into a specific form.

Example 5

Find the equation of the line through the points $(3, 7)$ and $(4, -2)$ in the form $y = mx + c$

$$m = \frac{(-2) - 7}{4 - 3}$$

$$= -9$$

So the equation is $y - 7 = -9(x - 3)$

$$y - 7 = -9x + 27$$

$$y = -9x + 34$$

First use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient.

Use $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (3, 7)$, or you could use the point $(4, -2)$ instead.

Expand the brackets and rearrange to the correct form.

Find the equation of the line through each pair of points.

Try It 5

- a** $(3, 7)$ and $(2, 9)$ **b** $(5, -1)$ and $(7, 5)$ **c** $(-3, -4)$ and $(7, 2)$

Lines with the same gradient are **parallel**. For example, $y = 5x + 2$ is parallel to $y = 5x - 7$, because the gradients are the same.

Example 6

The line l_1 has equation $2x + 6y = 5$. The line l_2 is parallel to l_1 and passes through the point $(1, -5)$. Find the equation of l_2 in the form $ax + by + c = 0$ where a , b and c are integers.

$$l_1: 2x + 6y = 5 \Rightarrow 6y = 5 - 2x$$

$$\Rightarrow y = \frac{5}{6} - \frac{2}{6}x$$

The gradient of l_1 is $-\frac{2}{6}$ which simplifies to $-\frac{1}{3}$

Therefore the gradient of l_2 is $-\frac{1}{3}$

So the equation of l_2 is $y - (-5) = -\frac{1}{3}(x - 1)$

$$\Rightarrow y + 5 = -\frac{1}{3}(x - 1)$$

$$\Rightarrow -3y - 15 = x - 1$$

$$\Rightarrow x + 3y + 14 = 0$$

Rearrange to make y the subject so you can see what the gradient is.

Since l_1 and l_2 are parallel.

Use $y - y_1 = m(x - x_1)$ to write the equation of l_2

Multiply both sides by -3 so that all coefficients are integers.

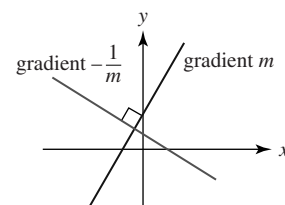
Rearrange to the correct form.

The line l_1 has equation $3x - 2y = 8$. A second line, l_2 is parallel to l_1 and passes through the point $(3, -2)$. Find the equation of l_2 in the form $ax + by + c = 0$ where a , b and c are integers.

Try It 6



Lines that meet at a right angle are **perpendicular**. The gradients of two perpendicular lines multiply to give -1 . For example, a line with gradient 5 is perpendicular to a line with gradient $-\frac{1}{5}$ since $5 \times \left(-\frac{1}{5}\right) = -1$



If the gradient of a line is m then the gradient of a perpendicular line is $-\frac{1}{m}$ since $m \times \left(-\frac{1}{m}\right) = -1$

Key point

Example 7

Decide whether or not each line is parallel or perpendicular to the line $y = 4x - 1$

- a** $2x + 8y = 5$ **b** $20x + 5y = 2$ **c** $16x - 4y = 5$

First note that the gradient of $y = 4x - 1$ is 4

a $2x + 8y = 5 \Rightarrow 8y = 5 - 2x$

$\Rightarrow y = \frac{5}{8} - \frac{1}{4}x$

$4 \times \left(-\frac{1}{4}\right) = -1$ so this line is perpendicular to $y = 4x - 1$

b $20x + 5y = 2 \Rightarrow 5y = 2 - 20x$

$\Rightarrow y = \frac{2}{5} - 4x$

The gradient is -4 so this line is neither parallel nor perpendicular to $y = 4x - 1$

c $16x - 4y = 5 \Rightarrow 4y = 16x - 5$

$\Rightarrow y = 4x - \frac{5}{4}$

The gradient is 4 so this line is parallel to $y = 4x - 1$

Rearrange to make y the subject.

The gradient is $-\frac{1}{4}$

Since the product of the gradients is -1

Rearrange to make y the subject.

Decide whether or not each line is parallel or perpendicular to the line $y = 4 - 3x$

Try It 7

- a** $3x + 6y = 2$ **b** $5x - 15y = 7$ **c** $18x + 6y + 5 = 0$



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Example 8

The line l_1 has equation $7x + 4y = 8$. The line l_2 is perpendicular to l_1 and passes through the point $(7, 3)$. Find the equation of l_2 in the form $ax + by + c = 0$ where a , b and c are integers.

$$l_1: 7x + 4y = 8 \Rightarrow 4y = -7x + 8$$

$$\Rightarrow y = -\frac{7}{4}x + 2$$

So the gradient of l_1 is $-\frac{7}{4}$ and the gradient of l_2 is $\frac{4}{7}$.

$$\text{So the equation of } l_2 \text{ is } y - 3 = \frac{4}{7}(x - 7)$$

$$\Rightarrow 7y - 21 = 4(x - 7)$$

$$\Rightarrow 7y - 21 = 4x - 28$$

$$\Rightarrow 4x - 7y - 7 = 0$$

Rearrange to the correct form.

Rearrange to make y the subject so you can see what the gradient is.

$$\text{Since } \left(-\frac{7}{4}\right) \times \frac{4}{7} = -1$$

Use $y - y_1 = m(x - x_1)$ to write the equation of l_2 .

Multiply both sides by 7 so that all coefficients are integers.



The line l_1 has equation $4x + 6y = 3$. A second line, l_2 is perpendicular to l_1 and passes through the point $(-1, 5)$. Find the equation of l_2 in the form $ax + by + c = 0$ where a, b and c are integers.

The **perpendicular bisector** of a line segment passes through its midpoint at a right angle.

Example 9

Find the equation of the perpendicular bisector of the line segment joining $(3, -4)$ and $(9, -6)$

Midpoint is $\left(\frac{3+9}{2}, \frac{-4+(-6)}{2}\right) = (6, -5)$

Gradient of line segment is $\frac{-6 - (-4)}{9 - 3} = -\frac{2}{6} = -\frac{1}{3}$

So the perpendicular bisector has gradient $m = 3$

The equation of the perpendicular bisector is $y - (-5) = 3(x - 6)$
or $y = 3x - 23$

Use $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Use $m = \frac{y_2 - y_1}{x_2 - x_1}$

Since they are perpendicular and $3 \times \left(-\frac{1}{3}\right) = -1$

Use $y - y_1 = m(x - x_1)$

Find the equation of the perpendicular bisector of the line segment joining $(2, -3)$ and $(-12, 5)$



Four horizontal lines for writing, enclosed in a rounded rectangular border.



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1 Find the gradient of the line through each pair of points.

a $(3, 7)$ and $(2, 8)$

b $(5, 2)$ and $(-4, -6)$

c $(1.3, 4.7)$ and $(2.6, -3.1)$

d $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{3}{4}, \frac{2}{3}\right)$

e $(\sqrt{3}, 2)$ and $(2\sqrt{3}, 5)$

f $(3a, a)$ and $(a, 5a)$

2 Calculate the exact distance between each pair of points.

a (8, 4) and (1, 3)

b (-3, 9) and (12, -7)

c (5.9, 6.2) and (-8.1, 3.8)

d $\left(\frac{1}{5}, -\frac{1}{5}\right)$ and $\left(\frac{3}{5}, -\frac{4}{5}\right)$

e $(5, -3\sqrt{2})$ and $(2, \sqrt{2})$

f $(k, -3k)$ and $(2k, -6k)$

3 Find the coordinates of the midpoint of each pair of points.

a $(3, 9)$ and $(1, 7)$

b $(2, -4)$ and $(-3, -9)$

c $(2.1, 3.5)$ and $(6.3, -3.7)$

d $\left(\frac{2}{3}, -\frac{1}{2}\right)$ and $\left(-\frac{5}{3}, -\frac{3}{2}\right)$

e $(6\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, \sqrt{5})$

f $(m, 2n)$ and $(3m, -2n)$

4 Work out the gradient and the y -intercept of these lines.

a $y = 7x - 4$

b $y + 2x = 3$

c $x - y = 4$

d $3x+2y=7$

e $5x-2y=9$

f $5y-3x=0$

g $x+6y+3=0$

h $3(y-2)=4(x-1)$

5 Find the equation of the line through each pair of points.

a $(2, 5)$ and $(0, 6)$ _____

b $(1, -3)$ and $(2, -5)$ _____

c $(4, 4)$ and $(7, -7)$ _____

d $(8, -2)$ and $(4, -3)$ _____

e $(-3, -7)$ and $(5, 9)$ _____

f $(\sqrt{2}, -\sqrt{2})$ and _____
 $(3\sqrt{2}, 4\sqrt{2})$ _____

6 Which of these lines is either parallel or perpendicular to the line with equation $y = 6x + 5$?

a $2x + 12y + 3 = 0$ _____

b $18x + 3y = 2$

c $3x - \frac{1}{2}y + 5 = 0$

7 Which of these lines is either parallel or perpendicular to the line with equation $y = \frac{2}{3}x - 4$?

a $24x + 16y + 3 = 0$

b $6x + 9y + 2 = 0$

c $2x - 3y = 7$

8 Which of these lines is either parallel or perpendicular to the line with equation $6x+12y=1$?

a $2y=5-x$

b $9x=18y+4$

c $10x-5y+3=0$

In questions **9–13**, give your answers in the form $ax+by+c=0$ where a , b and c are integers.

9 The line l_1 has equation $y=5x+1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $(3, -3)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $(-4, 1)$

10 The line l_1 has equation $y = 3 + \frac{1}{2}x$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $(-1, 5)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $(6, 2)$

11 The line l_1 has equation $3x + y = 9$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $(8, -2)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $(-1, -1)$

12 The line l_1 has equation $6x + 5y + 2 = 0$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $(4, 0)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $(12, 3)$

13 The line l_1 has equation $6x - 2y = 1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $\left(\frac{1}{2}, 1\right)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $\left(-1, -\frac{1}{2}\right)$

14 Find the equation of the perpendicular bisector of the line segment joining each pair of points.

a $(5, -7)$ and $(-3, 5)$

b $(-5, -9)$ and $(5, 5)$

c $(-6, 2)$ and $(4, 12)$

d $(2, -7)$ and $(-1, 2)$

e $(-13, -5)$ and $(15, -12)$

15 Find the point of intersection between these pairs of lines.

a $y = 5x - 4$ and $y = 3 - 2x$

b $y = 8x$ and $y = 3x - 10$

c $y = 7x - 5$ and $y = -\frac{1}{2}x + 5$

d $y = \frac{1}{4}x + 7$ and $y = 5x - \frac{5}{2}$

16 Find the point of intersection between these pairs of lines.

a $2x + 3y = 1$ and $3x - y = 7$

b $3x - 2y = 4$ and $x + y = 8$

c $5x - 7y = 3$ and $2x + 8y = 3$

d $-8x + 5y = 1$ and $3x + 18y + 7 = 0$
