$$=-2\left[\left(x-\frac{5}{4}\right)^2-\frac{17}{16}\right]$$

$$=-2\left(x-\frac{5}{4}\right)^2+\frac{17}{8}$$

2a $x^2+14x=(x+7)^2-49$

So (–7, –49) is a minimum point

2b $x^2-18x+3=(x-9)^2-81+3$

$$=(x-9)^2-78$$

So (9, –78) is a minimum point

2c $x^2-9x=\left(x-\frac{9}{2}\right)^2-\frac{81}{4}$

So $\left(\frac{9}{2},-\frac{81}{4}\right)$ is a minimum point

2d $-x^2+4x=-\left[x^2-4x\right]$

$$=-\left[(x-2)^2-4\right]$$

$$=-(x-2)^2+4$$

So (2, 4) is a maximum point

2e $x^2+11x+30=\left(x+\frac{11}{2}\right)^2-\frac{121}{4}+30$

$$=\left(x+\frac{11}{2}\right)^2-\frac{1}{4}$$

So $\left(-\frac{11}{2},-\frac{1}{4}\right)$ is a minimum point

2f $-x^2+6x-7=-\left[x^2-6x+7\right]$

$$=-\left[(x-3)^2-9+7\right]$$

$$=-\left[(x-3)^2-2\right]$$

$$=-(x-3)^2+2$$

So (3, 2) is a maximum point

2g $2x^2+16x-5=2\left[x^2+8x-\frac{5}{2}\right]$

$$=2\left[(x+4)^2-16-\frac{5}{2}\right]$$

$$=2\left[(x+4)^2-\frac{37}{2}\right]$$

$$=2(x+4)^2-37$$

So (–4, –37) is a minimum point

2h $-3x^2+15x-2=-3\left[x^2-5x+\frac{2}{3}\right]$

$$=-3\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{2}{3}\right]$$

$$=-3\left[\left(x-\frac{5}{2}\right)^2-\frac{67}{12}\right]$$

$$=-3\left(x-\frac{5}{2}\right)^2+\frac{67}{4}$$

So $\left(\frac{5}{2},\frac{67}{4}\right)$ is a maximum point.

Try it 1E

1 $7x^2-4x-6=0$

$a=7$, $b=-4$, $c=-6$

$$x=\frac{-(-4)+\sqrt{(-4)^2-4\times7\times(-6)}}{2\times7}$$

$$=1.25$$

$$x=\frac{-(-4)-\sqrt{(-4)^2-4\times7\times(-6)}}{2\times7}$$

$$=-0.68$$

$x=1.25$ or $x=-0.68$

2 $kx^2-x+5=0$

$a=k$, $b=-1$, $c=5$

$b^2-4ac=(-1)^2-4\times k\times5$

$$=1-20k$$

One real solution so $b^2-4ac=0$

So $1-20k=0\Rightarrow k=\frac{1}{20}$

3 $x^2+3x-k=0$

$a=1$, $b=3$, $c=-k$

$b^2-4ac=3^2-4\times1\times(-k)$

$$=9+4k$$

Real solutions so $b^2-4ac\geq0$

So $9+4k\geq0\Rightarrow k\geq-\frac{9}{4}$

4 $kx^2-7x+1=0$

$a=k$, $b=-7$, $c=1$

$b^2-4ac=(-7)^2-4\times k\times1$

$$=49-4k$$

No real solutions so $b^2-4ac<0$

So $49-4k<0\Rightarrow k>\frac{49}{4}$

Bridging Exercise 1E

1a $7x^2+3x-8=0$

$a=7$, $b=3$, $c=-8$

$$x=\frac{-3+\sqrt{3^2-4\times7\times(-8)}}{2\times7}$$

$$=0.88$$

$x = \dfrac{-3-\sqrt{3^2-4\times7\times(-8)}}{2\times7}$

$= -1.30$

$x = 0.88$ or $x = -1.30$

1b $-x^2+4x-2=0$

$a=-1,\ b=4,\ c=-2$

$x = \dfrac{-4+\sqrt{4^2-4\times(-1)\times(-2)}}{2\times(-1)}$

$= 3.41$

$x = \dfrac{-4-\sqrt{4^2-4\times(-1)\times(-2)}}{2\times(-1)}$

$= 0.59$

$x = 3.41$ or $x = 0.59$

1c $x^2-12x+4=0$

$a=1,\ b=-12,\ c=4$

$x = \dfrac{-(-12)+\sqrt{(-12)^2-4\times1\times4}}{2\times1}$

$= 11.66$

$x = \dfrac{-(-12)-\sqrt{(-12)^2-4\times1\times4}}{2\times1}$

$= 0.34$

$x = 11.66$ or $x = 0.34$

2a $x^2-5x+7=0$

$a=1,\ b=-5,\ c=7$

So discriminant $= b^2-4ac$

$= (-5)^2-4(1)(7) = 25-28$

$= -3$

$-3 < 0$ so no real solutions.

2b $7-2x-3x^2=0$

$a=-3,\ b=-2,\ c=7$

So discriminant $= b^2-4ac$

$= (-2)^2-4(7)(-3) = 4+84$

$= 88$

$88 > 0$ so two (distinct) real solutions.

2c $4x^2-28x+49=0$

$a=4,\ b=-28,\ c=49$

So discriminant $= b^2-4ac$

$= (-28)^2-4(4)(49) = 784-784$

$= 0$

So one real solution (coincidental solutions).

3a $y=7x^2-5x+4$ since

$a=7,\ b=-5,\ c=4$

So $b^2-4ac = (-5)^2-4\times7\times4$

$= -87$

$-87 < 0$ so no real solutions and the curve has a $\cup$ shape

3b $y=-4x^2+12x-9$ since

$a=-4,\ b=12,\ c=-9$

So $b^2-4ac = 12^2-4\times(-4)\times(-9)$

$= 0$

so one real solution

3c $y=6x^2-x-15$ since

$a=6,\ b=-1,\ c=-15$

So $b^2-4ac = (-1)^2-4\times6\times(-15)$

$= 361$

$361 > 0$ so two real solutions

3d $y=-x^2+2x-4$ since

$a=-1,\ b=2,\ c=-4$

So $b^2-4ac = 2^2-4\times(-1)\times(-4)$

$= -12$

$-12 < 0$ so no real solutions and the curve has a $\cap$ shape

4a $3x^2+2x-k=0$

$a=3,\ b=2,\ c=-k$

$b^2-4ac = 2^2-4\times3\times(-k)$

$= 4+12k$

Exactly one solution so

$b^2-4ac=0 \Rightarrow 4+12k=0$

$k = -\dfrac{1}{3}$

4b $kx^2-x+4=0$

$a=k,\ b=-1,\ c=4$

$b^2-4ac = (-1)^2-4\times k\times4$

$= 1-16k$

Exactly one solution so

$b^2-4ac=0 \Rightarrow 1-16k=0$

$k = \dfrac{1}{16}$

4c $2x^2+5x+k-5=0$

$a=2,\ b=5,\ c=k-5$

$b^2-4ac=5^2-4\times2\times(k-5)$

$=65-8k$

Exactly one solution so

$b^2-4ac=0 \Rightarrow 65-8k=0$

$k=\frac{65}{8}$

5a $x^2+3x-3k=0$

$a=1,\ b=3,\ c=-3k$

$b^2-4ac=3^2-4\times1\times(-3k)$

$=9+12k$

Real solutions so

$b^2-4ac\geq0 \Rightarrow 9+12k\geq0$

$k\geq-\frac{3}{4}$

5b $kx^2-7x+4=0$

$a=k,\ b=-7,\ c=4$

$b^2-4ac=(-7)^2-4\times k\times4$

$=49-16k$

Real solutions so

$b^2-4ac\geq0 \Rightarrow 49-16k\geq0$

$k\leq\frac{49}{16}$

5c $-x^2+6x-k-2=0$

$a=-1,\ b=6,\ c=-k-2$

$b^2-4ac=6^2-4\times(-1)\times(-k-2)$

$=28-4k$

Real solutions so

$b^2-4ac\geq0 \Rightarrow 28-4k\geq0$

$k\leq7$

6a $5x^2-x+2k=0$

$a=5,\ b=-1,\ c=2k$

$b^2-4ac=(-1)^2-4\times5\times2k$

$=1-40k$

No real solutions so

$b^2-4ac<0 \Rightarrow 1-40k<0$

$k>\frac{1}{40}$

6b $-kx^2+4x+5=0$

$a=-k,\ b=4,\ c=5$

$b^2-4ac=4^2-4\times(-k)\times5$

$=16+20k$

No real solutions so

$b^2-4ac<0 \Rightarrow 16+20k<0$

$k<-\frac{4}{5}$

6c $6x^2-5x+3-2k=0$

$a=6,\ b=-5,\ c=3-2k$

$b^2-4ac=(-5)^2-4\times6\times(3-2k)$

$=-47+48k$

No real solutions so

$b^2-4ac<0 \Rightarrow -47+48k<0$

$k<\frac{47}{48}$

Try it 1F

1a $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{8-7}{4-1}$

$=\frac{1}{3}$

1b $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{6--2}{4-8}$

$=\frac{8}{-4}$

$=-2$

1c $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-7-7}{-4--8}$

$=-\frac{14}{4}$

$=-\frac{7}{2}$

2a $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(7-5)^2+(4-2)^2}$

$=\sqrt{2^2+2^2}$

$=2\sqrt{2}$