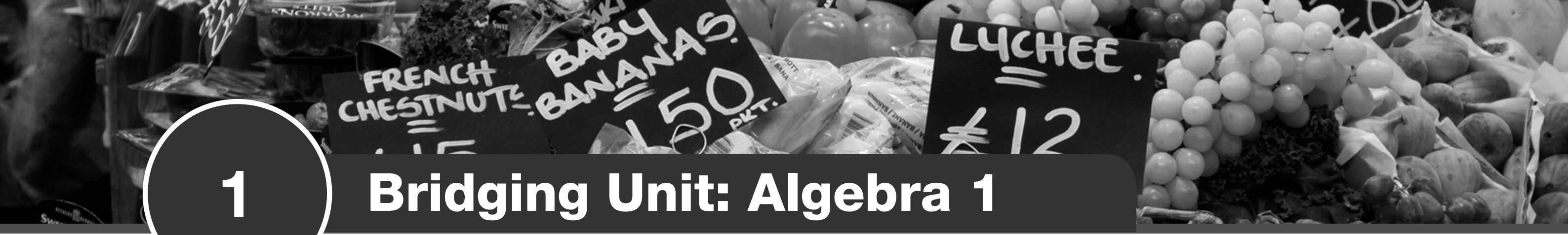

1 Bridging Unit: Algebra 1

Topic A: Indices and surds

Bridging to Ch1.2, 1.3

You can apply the rules of indices and surds to simplify algebraic expressions. The following expressions can be simplified in **index form**:

Key point

$x^a \times x^b = x^{a+b}$ $\quad x^a \div x^b = x^{a-b}$ $\quad (x^a)^b = x^{ab}$

Example 1

Simplify these expressions. **a** $2x^3 \times 3x^5$ **b** $12x^7 \div 4x^3$ **c** $(3x^5)^3$

a $2x^3 \times 3x^5 = 6x^{3+5}$

$= 6x^8$

Multiply the coefficients together and use $x^a \times x^b = x^{a+b}$

b $12x^7 \div 4x^6 = \dfrac{12x^7}{4x^6}$

$= 3x$

Since $\dfrac{12}{4} = 3$ and $x^a \div x^b = x^{a-b}$ so $\dfrac{x^7}{x^6} = x^1$ which we just write as x

c $(3x^5)^3 = 3^3(x^5)^3$

Both the 3 and the x^5 must be raised to the power 3

$= 27x^{15}$

Since $(x^a)^b = x^{ab}$

Try It 1

Simplify these expressions.

a $5x^3 \times 2x^7$ **b** $18x^9 \div 3x^2$ **c** $(2x^6)^4$ **d** $\left(\dfrac{x^3}{3}\right)^2$

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Roots can also be expressed using indices, such that the square root of x is written as $\sqrt{x} = x^{\frac{1}{2}}$

In general:

Key point

The nth root of x is written $\sqrt[n]{x} = x^{\frac{1}{n}}$, and this can be raised to a power to give $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Key point

A power of −1 indicates a reciprocal, so $x^{-1} = \frac{1}{x}$ and, in general, $x^{-n} = \frac{1}{x^n}$

Example 2

Evaluate each of these without using a calculator.

a $25^{0.5}$ **b** 6^{-2} **c** $8^{\frac{2}{3}}$

a $25^{0.5} = 25^{\frac{1}{2}}$

$= \sqrt{25}$ — Since a power of $\frac{1}{2}$ represents a square root.

$= 5$

b $6^{-2} = (6^2)^{-1}$

$= \frac{1}{6^2}$ — Since a power of −1 represents a reciprocal.

$= \frac{1}{36}$

c $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$ — Always calculate a root before a power.

$= 2^2$ — Since the cube root of 8 is 2

$= 4$

Try It 2

Evaluate each of these without a calculator.

a $36^{\frac{1}{2}}$ **b** $27^{\frac{2}{3}}$ **c** $64^{-0.5}$ **d** $\left(\frac{1}{2}\right)^4$

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Example 3

Write these expressions in simplified index form.

a $\sqrt[3]{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{2x}{\sqrt{x}}$

a $\sqrt[3]{x} = x^{\frac{1}{3}}$

b $\dfrac{2}{x^3} = 2x^{-3}$

c $\dfrac{2x}{\sqrt{x}} = \dfrac{2x}{x^{\frac{1}{2}}}$ — Since $\sqrt{x} = x^{\frac{1}{2}}$

$= 2x^{1-\frac{1}{2}}$ — Subtract the powers, remembering that $x = x^1$

$= 2x^{\frac{1}{2}}$

Try It 3

Write these expressions in simplified index form.

a $\sqrt[5]{x^2}$ **b** $\dfrac{3}{\sqrt{x}}$ **c** $\dfrac{3x^2}{\sqrt{x}}$ **d** $\dfrac{\sqrt{x}}{3x}$

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A **surd** is an irrational number involving a root, for example $\sqrt{2}$ or $\sqrt[3]{7}$.
You can multiply and divide surds using the rules:

Key point

$$\sqrt{a}\times\sqrt{b}=\sqrt{ab} \text{ and } \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$

An irrational number is a real number that cannot be written as a fraction $\frac{a}{b}$, where a and b are integers with $b \neq 0$

You can simplify surds by finding square-number factors, for example $\sqrt{12}=\sqrt{4}\sqrt{3}=2\sqrt{3}$. It may also be possible to simplify expressions involving surds by collecting like terms or by **rationalising the denominator**. Rationalising the denominator means rearranging the expression to remove any roots from the denominator.

Key point

To rationalise the denominator, multiply both the numerator and denominator by a suitable expression:

$\frac{1}{\sqrt{a}}\times\frac{\sqrt{a}}{\sqrt{a}}=\frac{\sqrt{a}}{a}$, (multiply numerator and denominator by $\sqrt{a}$)

$\frac{1}{a+\sqrt{b}}\times\frac{a-\sqrt{b}}{a-\sqrt{b}}=\frac{a-\sqrt{b}}{a^2-b}$, (multiply numerator and denominator by $a-\sqrt{b}$)

$\frac{1}{a-\sqrt{b}}\times\frac{a+\sqrt{b}}{a+\sqrt{b}}=\frac{a+\sqrt{b}}{a^2-b}$, (multiply numerator and denominator by $a+\sqrt{b}$)

Example 4

Simplify these expressions without using a calculator.

a $\sqrt{18}+5\sqrt{2}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{2}{1-\sqrt{5}}$

a $\sqrt{18}=\sqrt{9}\sqrt{2}$ — 9 is a square-number factor of 18 so you can simplify $\sqrt{18}$

$=3\sqrt{2}$

Therefore $\sqrt{18}+5\sqrt{2}=3\sqrt{2}+5\sqrt{2}$

$=8\sqrt{2}$ — Collect like terms.

b $\frac{6}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}$ — Rationalise the denominator by multiplying numerator and denominator by $\sqrt{3}$

$=\frac{6\sqrt{3}}{3}$

$=2\sqrt{3}$ — Since 6 ÷ 3 = 2

c $\frac{2}{1-\sqrt{5}}=\frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$ — Rationalise the denominator by multiplying numerator and denominator by $1+\sqrt{5}$

$=\frac{2(1+\sqrt{5})}{-4}$ — $(1-\sqrt{5})(1+\sqrt{5})=1-\sqrt{5}+\sqrt{5}-5 = 1-5=-4$

$=-\frac{1}{2}(1+\sqrt{5})$

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Try It 4

Simplify these expressions without using a calculator.

a $3\sqrt{28}-\sqrt{7}$ **b** $\frac{4}{\sqrt{3}}$ **c** $\frac{3}{1+\sqrt{2}}$ **d** $\frac{\sqrt{5}}{\sqrt{5}-2}$

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Bridging Exercise Topic A

Bridging to Ch1.2, 1.3

1 Evaluate each of these without using a calculator.

a $49^{\frac{1}{2}}$

b $27^{\frac{1}{3}}$

c 5^{-1}

d $64^{-\frac{1}{3}}$

e $9^{\frac{3}{2}}$

f $16^{\frac{3}{4}}$

g $125^{-\frac{2}{3}}$

h $\left(\frac{1}{2}\right)^3$

i $\left(\frac{1}{9}\right)^{-2}$

j $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

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k $\left(\frac{9}{16}\right)^{-0.5}$

l $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

2 Simplify these expressions fully without using a calculator.

a $\sqrt{8}$

b $\sqrt{75}$

c $2\sqrt{24}$

d $3\sqrt{48}$

e $\sqrt{20}+\sqrt{5}$

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f $\sqrt{27}-\sqrt{12}$

g $5\sqrt{32}-3\sqrt{8}$

h $\sqrt{50}+3\sqrt{125}$

i $\sqrt{68}+3\sqrt{17}$

j $3\sqrt{72}-\sqrt{32}$

k $4\sqrt{18}-2\sqrt{3}$

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l $6\sqrt{5}+\sqrt{50}$

3 Simplify these expressions fully without using a calculator.

a $\frac{1}{\sqrt{7}}$

b $\frac{2}{\sqrt{8}}$

c $\frac{12}{\sqrt{3}}$

d $\frac{\sqrt{8}}{\sqrt{12}}$

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e $\frac{1}{1+\sqrt{3}}$

f $\frac{2}{1+\sqrt{2}}$

g $\frac{8}{1-\sqrt{5}}$

h $\frac{2}{\sqrt{5}-1}$

i $\frac{\sqrt{2}}{2+\sqrt{3}}$

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j $\frac{2\sqrt{3}}{\sqrt{6}-2}$

k $\frac{1+\sqrt{2}}{1-\sqrt{2}}$

l $\frac{3+\sqrt{5}}{\sqrt{5}-3}$

4 Expand the brackets and fully simplify each expression.

a $(1+\sqrt{2})(3+\sqrt{2})$

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b $(1+\sqrt{2})(3-\sqrt{2})$

c $(1-\sqrt{2})(3+\sqrt{2})$

d $(1-\sqrt{2})(3-\sqrt{2})$

e $(\sqrt{3}+2)(4+\sqrt{3})$

f $(\sqrt{3}+2)(4-\sqrt{3})$

g $(\sqrt{3}-2)(4+\sqrt{3})$

h $(\sqrt{3}-2)(4-\sqrt{3})$

i $(\sqrt{6}+1)(\sqrt{2}+3)$

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j $(\sqrt{6}+1)(\sqrt{2}-3)$

k $(\sqrt{6}-1)(\sqrt{2}+3)$

l $(\sqrt{6}-1)(\sqrt{2}-3)$

5 Write each of these expressions in simplified index form.

a $x^3 \times x^7$

b $7x^5 \times 3x^6$

c $5x^4 \times 8x^7$

d $x^8 \div x^2$

e $8x^7 \div 2x^9$

f $3x^8 \div 12x^7$

g $(x^5)^7$

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h $(x^2)^{-5}$

i $(3x^2)^4$

j $(6x^5)^2$

k $\sqrt{x^3}$

l $\sqrt[4]{x^5}$

m $\frac{5\sqrt{x}}{x}$

n $2x\sqrt{x}$

o $\frac{x^2}{3\sqrt{x}}$

p $x^3(x^5-1)$

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q $x^3(\sqrt{x}+2)$

r $\frac{x+2}{x^3}$

s $\frac{\sqrt{x}+3}{x}$

t $\frac{(3-x^3)}{\sqrt{x}}$

u $(\sqrt{x}+3)^2$

v $\frac{3+\sqrt{x}}{x^2}$

w $\frac{1-x}{2\sqrt{x}}$

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x $\dfrac{\sqrt{x}+2}{3x^3}$

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