FURTHER MATHEMATICS

Holiday Work – Introduction

Congratulations on choosing Maths and Further Maths as two of your A level subjects, we trust you will enjoy the course and find it to be interesting and rewarding.

As with most subjects A Level Maths is very different from GCSE. Techniques you have learnt, especially in Algebra, will only be a part, but a vital one, of solving a problem. We find you tend to get a bit rusty over the long summer break so we ask that, towards the end of the holidays, you read through the attached notes and examples, and attempt the questions, to get your brain back into gear. You may find it helpful to look at your GCSE notes, or other resources such as MyMaths (www.mymaths.co.uk login: **pates** password: **proof).**

We would expect you to hand in your work on the questions on the first maths lesson you have. We will have a simple diagnostic test on Algebra at the end of the second week and feedback from your holiday work will prove very useful for revision.

We expect an excellent standard of written work. Your work should be precise, accurate and well structured.

If you cannot remember how to do a question, please don't worry, but you will need to be proactive. Use the Mymaths website shown above and do talk to your teacher as soon as possible Feel free to talk to your friends about concepts you are less confident on, however, do not just copy. You need to understand the concepts involved. Look out for the advertised 'maths support lesson', which you will be able to pop along to in the first week to ask any further questions.

By signing up for Further Maths, we correctly assume that you will be motivated in this subject. To this end, we have also included a couple of topics in this document that you will never have met. **The expectation is that you do some research and work out for yourselves how to do them.**

You may like to find more challenging ideas by looking at the NRICH website run by Cambridge University at www.nrich.maths.org. The epsilon section is particularly good to have a go at. This is designed for students who are particularly confident and in need of some extension work.

Mr R Esswood
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Holiday Work – Notes and examples

1) Simplifying Algebraic Expressions:

Add/subtract like terms (with same letter part) e.g.
$x+2x=3x$, x^2+2x cannot be simplified

Use laws of indices to multiply/divide powers e.g.
$3x^5\times 4x^3=12x^8$, $6x^5y^2\div 3x^2y=2x^3y$

Multiplying a bracket by a number – remember to multiply all terms, and take care where there is a negative multiplier e.g.
$2x(x+y-3z)=2x^2+2xy-6xz$, $-5(2x-3y)=-10x+15y$

Multiplying brackets together e.g.

$$(x+5)(x-2)=x^2-2x+5x-10=x^2+3x-10$$
$$(2x-7)(3y-4)=6xy-8x-21y+28$$
$$(x+8)(x-8)=x^2-8x+8x-64=x^2-64 \text{ (difference of two squares)}$$
$$(x+3)^2=(x+3)(x+3)=x^2+6x+9$$

Factorising e.g.

Common factors: $5x-20y-10=5(x-4y-2)$, $4x^3y+6xy^2=2xy(2x^2+3y)$
Two brackets
$x^2+13x+40=(x+5)(x+8)$ (where coefficient of x^2 is 1, look for numbers whose product is 40, and whose sum is 13)

If the coefficient of x^2 is not 1, you may be able to find the answer using a trial and improvement approach, or you may use this method :

$$5x^2+22x+8 \qquad \rightarrow 5\times 8=40, \textit{ find 2 numbers with product 40 and sum 22}$$
$$=5x^2+20x+2x+8 \qquad \rightarrow 20 \textit{ and } 2 \rightarrow 22x \textit{ becomes } 20x+2x$$
$$=5x(x+4)+2(x+4) \qquad \textit{Factorise pairs of terms}$$
$$=(5x+2)(x+4)$$

Difference of two squares

$$64x^2-81=(8x)^2-(9)^2=(8x-9)(8x+9)$$

2) Solving Quadratic equations:

Solving quadratic equations – remember there are usually two solutions

a) If no x term, can solve as for linear equations

$$4x^2 - 6 = 30$$
$$4x^2 = 36$$
$$x^2 = 9$$
$$x = \pm 3 \ \ (i.e\ x = 3\ and\ x = -3)$$

b) Factorising

$$2x^2 + 7x - 8 = 0$$
$$(2x-1)(x+4) = 0$$
$$2x - 1 = 0 \ \ or \ \ x + 4 = 0$$
$$x = \tfrac{1}{2} \ \ or \ \ x = -4$$

c) The quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, *where* $ax^2 + bx + c = 0$

$$6x^2 - 9x + 2 = 0 \rightarrow a = 6, b = -9, c = 2$$
$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4\times 6\times 2}}{2\times 6} = \frac{9 \pm \sqrt{33}}{12}$$
$$x = 1.23\ or\ 0.27 \ \ (correct\ to\ 2 decimal\ places)$$

The expression under the square root in the formula ($b^2 - 4ac$) is known as the **discriminant**. Do some research to find out what information this value gives you about the equation.

In the past, when the discriminant has been negative, you may have thought there are no roots to the quadratic equation. In fact the truth is that there are no REAL roots, but there are some IMAGINARY roots. The section of maths dealing with this concept is known as **COMPLEX NUMBERS**. Do some research on how to find the imaginary roots of quadratic equations, and use this research to help you to answer the questions given.

d) Completing the square

$$x^2 - 2x = 5$$
$$(x-1)^2 - 1 = 5$$
$$(x-1)^2 = 6$$
$$x - 1 = \pm\sqrt{6}$$
$$x = 1 \pm \sqrt{6} \rightarrow x = 3.45\ or\ x = -1.45$$

Your knowledge on solving quadratic equations can also be applied to solving quadratic trig equations. Again, we have set some questions on this, your task is to work out how to solve them even if you have never met them before.

3) Solving simple equations and inequalities:

Do the same to both sides, make unknown term positive. If you multiply or divide an inequality by a negative number, you need to reverse the sign.

4) Simultaneous equations:

Linear simultaneous equations may be solved by multiplying (if necessary) then adding or subtracting to eliminate one of the variables

$$\left.\begin{array}{l}2x+5y=19\\8x+3y=25\end{array}\right\}\rightarrow\left.\begin{array}{l}8x+20y=76\\8x+3y=25\end{array}\right\}\rightarrow 17y=51\rightarrow y=3$$

$$\text{and}\quad 2x+5\times 3=19\rightarrow 2x=4\rightarrow x=2$$

Linear simultaneous equations may also be solved by substitution

$$\left.\begin{array}{l}2x+3y=4\\x=2y+9\end{array}\right\}\rightarrow 2(2y+9)+3y=4\rightarrow 4y+18+3y=4$$

$$7y=-14\rightarrow y=-2$$

$$x=2\times -2+9\rightarrow x=5$$

Quadratic simultaneous equations generally have two solutions. You will need to use substitution to solve them.

$$\left.\begin{array}{l}x^2-2y^2=4\\x-y=2\end{array}\right\}\rightarrow\left.\begin{array}{l}x^2-2y^2=4\\x=y+2\end{array}\right\}\rightarrow (y+2)^2-2y^2=4$$

$$y^2+4y+4-2y^2=4$$

$$4y-y^2=0$$

$$y(4-y)=0\rightarrow y=0\ \text{ and }\ y=4$$

$$\text{Solutions}\quad y=0, x=2\ \text{ and }\ y=4, x=6$$

Holiday Work – Exercise: Write your solutions on A4 paper.

1. Simplify the following expressions :

a) $3(2x-5)-5(x+4)$ b) $2x(x+y)+y(4x-y+3)$ c) $\frac{6z^3 \times 4z^5}{3z^2 \times z^4}$

2. Expand and simplify where possible :

a) $(p+6)(p+2)$ b) $(2q-3)(2q+3)$ c) $(3r-5)(4r^2-1)$

3. Factorise :

a) $6a^3+4a^2+10a^4$ b) $b^2-7b+12$ c) $5c^2+16c+3$ d) $6d^2+7d-20$ e) e^2-25

4. Simplify these algebraic fractions :

a) $\frac{x}{2}-\frac{x}{3}$ b) $\frac{3}{x}+\frac{1}{2x}$ c) $\frac{2}{x+2}+\frac{3}{x-1}$

5. Use factorisation to simplify these expressions :

a) $\frac{x^2+3x+2}{x^2-4}$ b) $\frac{2x^2-4x+2}{6x} \times \frac{3x^2+3x}{x^2-1}$

6. Write in completed square form : a) x^2+2x+5 b) y^2-6y-1

7. Solve the following equations :

a) $5-2(m-1)=3m-8$ b) $2(1-3n)=4(2-n)$

8. Solve the following quadratic equations, giving answers to 2dp where appropriate:

a) $x^2+8x-20=0$ b) $y^2-49=0$ c) $3z^2+7z+2=0$ d) $p^2+8p+11=0$

e) $20-q-3q^2=0$ f) $x^2-4x+13=0$ g) $x^2-10x+40=0$

9. Solve for θ , giving your answers correct to 1 decimal place, $0 \leq \theta \leq 360$:

a) $2sin^2\theta + sin\theta - 1 = 0$ b) $3cos^2\theta - cos\theta = 0$

10. Solve :

a) $x(3x+7) = 2(9+2x)$ b) $\dfrac{9+x^2}{3x} = 2$ c) $\dfrac{6x}{x-2} = x+8$

11. a) Solve $\dfrac{1}{p} + \dfrac{1}{p-1} = -2$

b) Explain why the equation $\dfrac{x}{6} + \dfrac{5}{3x} = 1$ has no solutions.

12. Solve these pairs of linear simultaneous equations :

a) $x + y = 8$, $2x - y = 1$ b) $5x + 4y = 5$, $3x + 10y = 22$ c) $4p - 4q - 1 = 0$, $2q = 4p + 2$

13. Solve these pairs of simultaneous equations :

a) $x^2 + y^2 = 10$, $y = x + 4$ b) $2x^2 + xy = 18$, $x + y = 7$